Christ Church Grammar School

2019
TEST 2

MATHEMATICS METHODS Year 11

MARKING KEY

Time and marks available:

Calculator-Free

Reading time	3 minutes
Working time for this section:	30 minutes
Marks available:	**32 marks**

Calculator-Assumed

Working time for this section:	10 minutes
Marks available:	**10 marks**

Materials required/recommended:
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

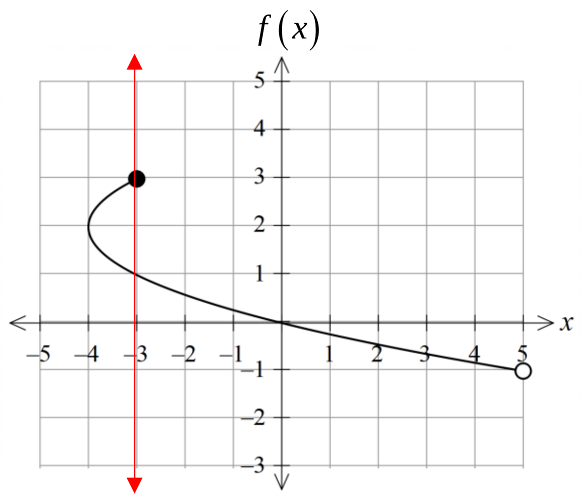
6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

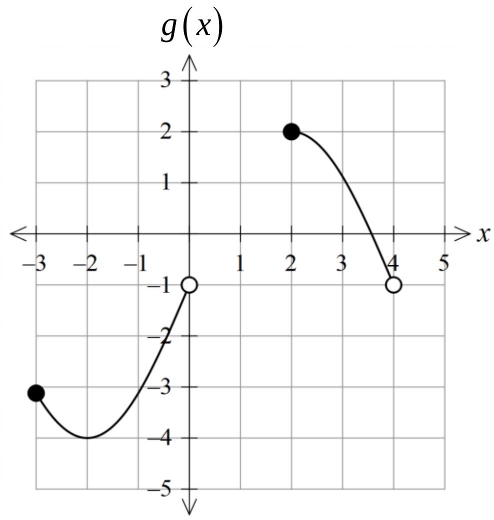
7. It is recommended that **you do not use pencil**, except in diagrams.

Calculator-Free Section (33 marks)

Question 1 (6 marks)

The graphs of relations f and g are shown below.





(a) Which of the relations f or g is NOT a function. (2 marks)
Explain.

Solution
Relation f is NOT a function, since there are instances where TWO ordered pairs have the same x coordinate e.g. $(-3,1)$ and $(-3,3)$. That is a vertical line can be drawn to intersect the graph of f in more than one point.
Specific behaviours
✓ states that f is NOT a function 1.1.28
✓ explains why f is not a function 1.1.28

(b) State the domain of relation f. (2 marks)

Solution
Range $D_f = \{x \mid -4 \le x < 5\}$
Specific behaviours
✓ indicates all real numbers from -4 to 5 1.1.24
✓ excludes the value $x = 5$ from the domain 1.1.24

(c) State the range of relation g. (2 marks)

Solution
Domain $R_g = \{y \mid -4 \le y \le 2,\ y \ne -1\}$
Specific behaviours
✓ indicates all real numbers from -4 to 2 1.1.24
✓ excludes the value $y = -1$ from the range 1.1.24

Question 2 **(6 marks)**

Solve exactly the following equations:

(a) $\dfrac{5}{x-3} = \dfrac{3}{x+4}$ (3 marks)

Solution
Multiplying each side by $(x-3)(x+4)$ obtains: $5(x+4) = 3(x-3)$ ... (1) i.e. $5x+20 = 3x-9$ ... (2) i.e. $2x=-29$ $\therefore$ $x=-\dfrac{29}{2}$ or $x=-14.5$
Specific behaviours
✓ multiplies both sides by $(x-3)(x+4)$ to obtain equation (1) ✓ expands correctly to obtain equation (2) or its equivalent ✓ solves correctly to obtain x

1.1.6

(b) $x(x-12) = -5$ (3 marks)

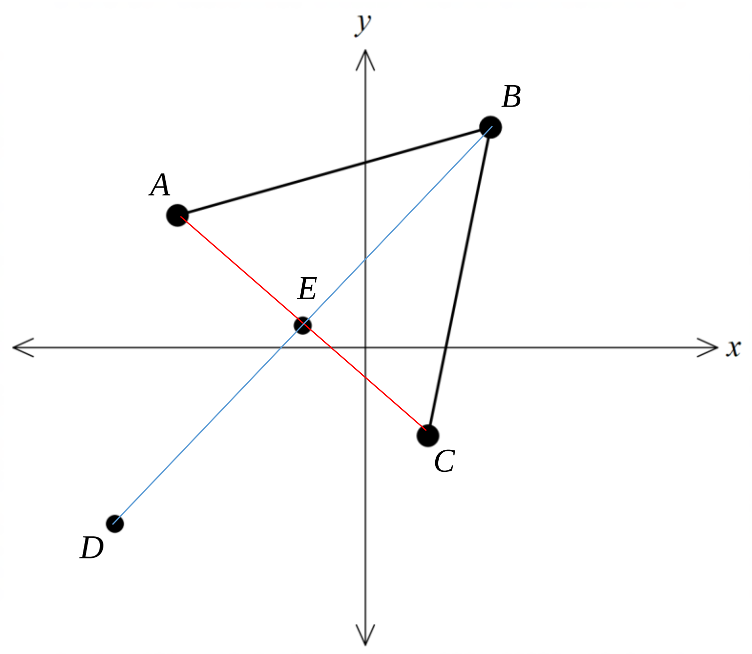
Solution
$\therefore$ $x^2-12x+5 = 0$ i.e. $(x-6)^2-36+5=0$ $\therefore$ $(x-6)^2=31$ $x-6 = \pm\sqrt{31}$ i.e. $x=6+\sqrt{31}$ or $x=6-\sqrt{31}$
Specific behaviours
✓ obtains the standard quadratic equation correctly ✓ performs the completion of square process correctly ✓ solves correctly to obtain $x=6\pm\sqrt{31}$

Alternative Solution
$\therefore$ $x^2-12x+5 = 0$ i.e. $x = \dfrac{-(-12)\pm\sqrt{(-12)^2-4(1)(5)}}{2(1)}$ using the Quadratic Formula i.e. $x = \dfrac{12\pm\sqrt{124}}{2} = \dfrac{12\pm2\sqrt{31}}{2} = 6\pm\sqrt{31}$ i.e. $x=6+\sqrt{31}$ or $x=6-\sqrt{31}$
Specific behaviours
✓ obtains the standard quadratic equation correctly ✓ substitutes correctly into the quadratic formula ✓ solves correctly to obtain $x=6\pm\sqrt{31}$

1.1.9

Question 3 **(10 marks)**

The graph indicates points $A\ (-3,3)$, $B\ (2,5)$ and $C\ (1,-2)$. Point D is positioned so that $ABCD$ is a parallelogram. Point E is the midpoint of both $\overline{AC}$ and $\overline{BD}$ since it is a property of a parallelogram that the diagonals bisect each other. The coordinates of E are $(-1,0.5)$.



(a) Determine the equation for $\overleftrightarrow{BC}$ in the form $y = mx + c$. (3 marks)

Solution
$m(\overline{BC}) = \dfrac{5-(-2)}{2-1} = 7$ Equation for $\overleftrightarrow{BC}$: $y-5=7(x-2)$ *i.e.* $y-5 = 7x-14$ $\therefore\ y = 7x-9$
Specific behaviours
✓ determines the gradient correctly ✓ forms the equation of the line correctly using point B or C ✓ writes the equation correctly in the form $y = mx + c$

1.1.5

(b) Using the coordinates of $E\ (-1,0.5)$, determine the coordinates for point D. (2 marks)

Solution
Let point D be (a,b) $\therefore\ -1 = \dfrac{2+a}{2}$ and $0.5 = \dfrac{5+b}{2}$ Solving gives $a=-4$ and $b=-4$ i.e. D is the point $(-4,-4)$.
Specific behaviours
✓ writes the equations relating the coordinates of E in terms of B and D correctly ✓ solves these equations correctly to determine point D

1.1.2

(b) Using the coordinates of $E\ \left(-1, 0.5\right)$, determine the coordinates for point D. (2 marks)

Alternative Solution
From $B \to E \quad \Delta x = -3$ and $\Delta y = -4.5$ So $E \to D$ will also have $\Delta x = -3$ and $\Delta y = -4.5$ i.e. D is the point $\left(-1-3,\ 0.5-4.5\right) = \left(-4,-4\right)$.
Specific behaviours
✓ determines the step sizes/changes in coordinates from $B \to E$ correctly ✓ determines point D correctly

1.1.2

Consider a line containing C and perpendicular to $\overleftrightarrow{AB}$.

(c) Determine the equation for this perpendicular line. (3 marks)

Solution
$m\left(\overline{AB}\right) = \dfrac{5-3}{2-(-3)} = \dfrac{2}{5} \qquad \therefore\ m\left(Perp\right) = -\dfrac{5}{2}$ since $m_1 m_2 = -1$ Equation for Perpendicular containing C : $\quad y-(-2) = -\dfrac{5}{2}(x-1)$ $i.e.\quad 2\left(y+2\right) = -5\left(x-1\right)$ $\therefore\ 5x+2y=1$ OR $\quad y = 0.5-2.5x \quad$ OR $\quad y = \dfrac{1-5x}{2}$
Specific behaviours
✓ determines the gradient of $\overleftrightarrow{AB}$ correctly ✓ determines the perpendicular gradient correctly using $m_1 m_2 = -1$ ✓ forms the equation correctly containing point C

1.1.5

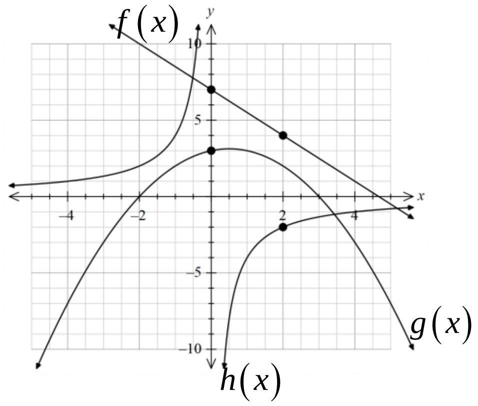
(d) Show that $ABCD$ is NOT a rectangle. (2 marks)

Solution
We need to show that $s\angle ABC \neq 90^o$ i.e. $m\left(\overleftrightarrow{AB}\right)m\left(\overleftrightarrow{BC}\right) \neq -1$ $m\left(\overline{AB}\right) = \dfrac{2}{5} \qquad m\left(\overline{BC}\right) = 7$ Since $\dfrac{2}{5}\times 7 \neq -1$ then $ABCD$ is NOT a rectangle.
Specific behaviours
✓ states or infers that if $ABCD$ is a rectangle then sides are at right angles ✓ states or concludes that the product of adjacent gradients $\neq -1$

1.1.5

Question 4 **(7 marks)**

The diagram below shows the graphs of functions $f(x)$, $g(x)$ and $h(x)$.



Determine the defining rules for function:

(a) $f(x)$. (2 marks)

Solution
Vertical intercept $(0,7)$ Gradient $m=-1.5$ $\therefore$ $f(x) = -1.5x+7$
Specific behaviours
✓ calculates the correct gradient ✓ writes the equation of the line correctly (using correct notation)

1.1.4

(b) $g(x)$. (3 marks)

Solution
x intercepts are $x=-2$ and $x=3$ $\therefore$ $g(x) = k(x+2)(x-3)$ using Factor Form Using $(0,3)$ $3 = k(0+2)(0-3)$ Solving gives $k=-0.5$ i.e. $\therefore$ $g(x) = -0.5(x+2)(x-3) = -0.5x^2+0.5x+3$
Specific behaviours
✓ uses quadratic factor form $g(x)=k(x-a)(x-b)$ ✓ uses $a=-2$ and $b=3$ ✓ uses a known ordered pair to correctly deduce the value of k (dilation factor)

1.1.8, 1.1.10

(c) $h(x)$. (2 marks)

Solution
Graph suggests the defining rule $h(x) = \frac{k}{x}$ Using $(2,-2)$ $-2 = \frac{k}{2}$ i.e. $k=-4$ $\therefore$ $h(x) = -\frac{4}{x}$
Specific behaviours
✓ uses the form $h(x) = \frac{k}{x}$ ✓ writes the reciprocal defining rule correctly

1.1.14

Question 5 **(3 marks)**

The graph of $y = kx^2 + 4x + k$ has no x intercepts.
Determine the value(s) of the constant k.

Solution
The intersection with the x-axis occurs when $kx^2 + 4x + k = 0$. Hence this means that $kx^2 + 4x + k = 0$ has NO solutions. $\therefore\ \Delta < 0$ $\therefore\ (4)^2 - 4(k)(k) < 0$ i.e. $16 - 4k^2 < 0$ i.e. $k^2 > 4$ $\therefore\ k > 2$ *or* $k < -2$ for no x intercepts.
Specific behaviours
✓ states that the discriminant must be negative for no solutions ✓ forms the correct expression for the discriminant ✓ solves correctly for the value for k

1.1.11

Calculator-Assumed Section (10 marks)

Question 6 (4 marks)

The pressure P, measured in kPa, exerted by a certain mass of gas at room temperature is inversely proportional to its volume V, measured in $litres$.

This particular amount of gas exerts a pressure of $2.75\ kPa$ when its volume is $4.5\ litres$.

(a) Express the relationship between the pressure P and the volume V. (2 marks)

Solution
Since $P \propto \frac{1}{V}$ we can write $P = \frac{k}{V}$.
Substituting $V = 4.5$ and $P = 2.75$ then $2.75 = \frac{k}{4.5}$ $\therefore\ k = 12.375$
Hence $P = \frac{12.375}{V}$ OR $PV = 12.375$
Specific behaviours
✓ expresses pressure in terms of the reciprocal of volume ✓ determines the reciprocal rule correctly (in any form)

1.1.13

(b) If the volume of this gas is reduced by $0.7\ litres$, determine the increase in the pressure of the gas, correct to 2 decimal places. (2 marks)

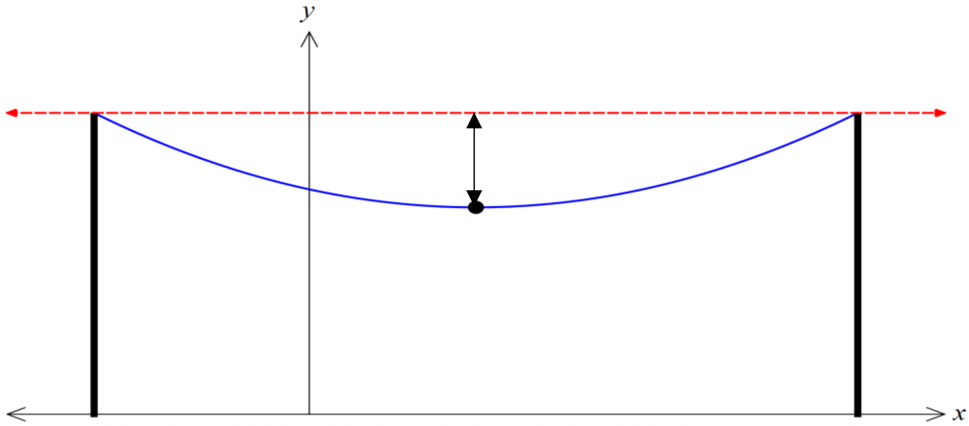
Solution
Using $V = 4.5 - 0.7 = 3.8$ litres
Then $P = \frac{12.375}{(3.8)} = 3.2565....$ kPa
$\therefore\ \Delta P = 3.265.. - 2.75$ Hence the pressure will increase by 0.51 kPa. $= 0.506...$
Specific behaviours
✓ substitutes the correct volume value ✓ determines the correct increase in pressure

1.1.13

Question 7 (6 marks)

A high voltage power line is supported by support towers that are each 6.7 m in height. The 'sag' in the power line is defined to be the vertical distance the power line is below 6.7 m

The height of the power line between the towers is modelled by the quadratic function $y = 0.004x^2 - 0.08x + 5$ as shown below.



(a) Determine the distance between the support towers, correct to the nearest 0.01 metres (3 marks)

Solution
Require $y = 6.7$ i.e. solve $0.004x^2 - 0.08x + 5 = 6.7$ From CAS $x = -12.913...$, $x = 32.913...$ Hence the distance between the support towers $= 32.913... - (-12.913..)$ $= 45.826...$ $\therefore$ Distance between towers is 45.83 metres
Specific behaviours
✓ forms the quadratic equation correctly for $y = 6.7$ ✓ solves the quadratic equation correctly ✓ determines the distance between the towers correctly to 0.01 metres

1.1.9 and 1.1.12

(b) Determine the maximum sag in the power line, correct to the nearest 0.01 metres. (3 marks)

Solution
The greatest sag will occur at the turning point for the parabolic shaped power line. From CAS this is at the point $(10,\ 4.6)$ OR $y = 0.004(x-10)^2 + 4.6$ Hence the greatest sag $= 6.7 - 4.6$ $= 2.1$ i.e. the greatest sag is 2.10 metres.
Specific behaviours
✓ uses the idea of determining the turning point of the height function ✓ determines the minimum function value (minimum height) ✓ determines the maximum sag correctly

1.1.12